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A Solution of Mathematical Multi-Objective Green Transportation Problems Under the Fermatean Fuzzy Environment

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Abstract

This paper presents a mathematical framework for solving the Green Transportation Problem (TP) under uncertainty using Fermatean Fuzzy Parameters (FFP). Focusing on a Multi-Objective Transportation Problem (MOTP), the model aims to simultaneously minimize transportation cost, travel time, and carbon emissions—key metrics in sustainable logistics. All parameters, including costs, supply, and demand, are treated as Fermatean fuzzy to capture real-world ambiguity and uncertainty better. We introduce a New Fermatean Fuzzy Score Function (NFFSF) that converts fuzzy values into crisp equivalents to handle these fuzzy parameters. A Fermatean Fuzzy Programming Approach (FFPA) is applied to derive compromise solutions that balance economic efficiency with environmental responsibility. A numerical example illustrates the practicality and effectiveness of the proposed method in supporting eco-friendly and optimized transportation planning. This approach provides a robust decision-making tool for achieving sustainability goals in complex and uncertain transportation environments.

Keywords: Fermatean fuzzy parameters, Fermatean fuzzy transportation problems, Multi-objective optimization, Mathematical modeling.

1 | Introduction

The Transportation Problem (TP) is a foundational optimization problem in operations research, widely used in logistics and supply chain management to determine the most cost-effective way to distribute goods from multiple suppliers to multiple consumers while meeting supply and demand constraints [1], [2]. Traditionally, this problem has focused on minimizing transportation costs—commonly called the traditional TP [3]. However, the growing demand for sustainable and eco-efficient logistics has led to the emergence of the

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Multi-Objective Transportation Problem (MOTP), where multiple conflicting goals—such as minimizing transportation time, carbon emissions, and resource usage—are optimized simultaneously [4], [5].

In modern logistics networks, uncertainty in transportation parameters such as cost, time, supply, and demand is inevitable, especially under volatile economic and environmental conditions [1]. Fuzzy set theory has proven to be an effective tool for modeling such vagueness. Traditional fuzzy approaches, such as intuitionistic and Pythagorean fuzzy sets, have certain limitations when capturing higher degrees of uncertainty [6], [7]. Fermatean Fuzzy Sets (FFSs) have been introduced to overcome these limitations, offering a more flexible framework for representing uncertain data through extended truth, falsity, and indeterminacy memberships [3]. Transportation systems today face immense challenges of climate change, fuel consumption, and emissions reduction. This shift has fueled the development of green transportation models that incorporate environmental impact metrics, such as carbon footprint, into classical logistics optimization. [8] pressure to become greener, faster, and more reliable in response to the pressing global crisis. Integrating these concerns into the MOTP framework provides a more comprehensive basis for real-world decision-making, where minimizing operational cost alone is no longer sufficient [1]. There is a growing recognition that multi-objective optimization must include sustainability goals alongside traditional efficiency metrics. Despite the advances in fuzzy modeling, few studies have directly addressed multi-objective green TPs using Fermatean fuzzy logic. Existing methods often simplify the complexity of environmental parameters or fail to transform fuzzy values into actionable insights effectively. The present study introduces a comprehensive solution framework to address this gap by developing a Fermatean Fuzzy Programming Approach (FFPA) and an New Fermatean Fuzzy Score Function (NFFSF). These tools enable more accurate modeling of uncertain, sustainability-related parameters and facilitate deriving compromise optimal solutions that balance cost, time, and environmental impact. This research aims to contribute a robust and flexible decision-making model that aligns with the evolving demands of green supply chain practices.

The remainder of this paper is structured as follows: Section 2 discusses the literature review of the proposed problem. Section 3 introduces the fundamental definitions, theorems, and basic arithmetic operations associated with FFSs, which serve as the mathematical foundation for the proposed model, presents the formulation of the traditional TP and the MOTP, and demonstrates how these models are transformed into their crisp equivalents using the NFFSF. Section 3 proposes a mathematical modeling approach based on the FFPA, which is designed to handle multi-objective optimization under uncertainty. Section 4 outlines the methodology for solving the proposed MOTP model within the Fermatean fuzzy environment. Section 5 provides a detailed numerical example to illustrate the applicability and effectiveness of the proposed approach. Finally, Section 6 concludes the paper by summarizing the key findings and suggesting possible directions for future research.

2 | Literature Review

The TP has evolved from single-objective formulations to multi-objective extensions that align with real-world complexities. The MOTP includes diverse objectives such as minimizing cost, time, deterioration of goods, inventory levels, emissions, congestion, or maximizing customer satisfaction and vehicle utilization [9], [10]. Solutions to these problems often require sophisticated techniques like goal programming, multi-objective genetic algorithms, interactive evolutionary methods, and fuzzy programming [11], [12]. In recent years, the focus on environmental sustainability has further expanded the scope of MOTP, giving rise to green TPs where environmental objectives such as reducing carbon footprint, optimizing fuel efficiency, and minimizing pollutant emissions are prioritized alongside economic goals. This shift reflects a broader logistics and supply chain optimization trend toward eco-efficient decision-making. Incorporating such green objectives introduces additional layers of complexity, as trade-offs must be made between cost-effectiveness and environmental impact [13]. As a result, traditional linear and deterministic models often fall short in addressing the uncertainties and interdependencies among these competing goals. This has created a pressing need for more flexible and robust optimization frameworks, particularly those capable of capturing vagueness and ambiguity inherent in real-world data, where fuzzy and hybrid intelligent systems have shown considerable promise.

Uncertainty in transportation parameters has long been addressed using fuzzy set theory. Early methods utilized intuitionistic and Pythagorean fuzzy sets, but these had limitations in handling high uncertainty and conflicting objectives. FFSs, introduced by Senapati and Yager [3], extended these theories by allowing the square sum of truth and falsity membership degrees to be less than or equal to one. This made them particularly well-suited for modeling ambiguous data in complex decision-making environments. Senapati and Yager [3], [6] proposed operations and ranking methods for FFSs, including a score function for evaluating Fermatean fuzzy numbers. Sharma et al. [14] applied Fermatean Fuzzy Parameters (FFP) in optimizing transportation systems and presented algorithms based on Fermatean fuzzy logic. Akram et al. [7] introduced an extended Data Envelopment Analysis (DEA) approach for solving fuzzy TPs and proposed Trapezoidal FFPA to convert fuzzy problems into crisp form. Similarly, Ali and Javaid [8] developed a model using a novel score function and expected value method to tackle multi-objective fuzzy transportation issues.

Sahoo [15], [16] emphasized using Fermatean fuzzy models in volatile logistics environments. His studies demonstrated how FFP and score-based models can pragmatically address uncertainty in transportation costs and supply-demand variability. Bouraima et al. [17] and Chaudhary et al. [5] proposed fuzzy frameworks for green urban transportation planning, showing that fuzzy logic models, including Fermatean variants, are gaining traction in sustainable decision support systems. While several studies have successfully applied fuzzy logic to TPs, there remains a significant gap in addressing green MOTPs, specifically under Fermatean Fuzzy Environments (FFEs). Most existing works focus either on cost or single-objective optimization or use less flexible fuzzy frameworks that cannot fully capture the complex uncertainty of real-world environmental factors. This paper addresses that gap by:

I. Developing a multi-objective green transportation model under a Fermatean fuzzy framework.

II. Proposing an NFFSF for transforming fuzzy parameters into crisp values.

III. Applying an FFPA to derive compromise solutions that balance cost, time, and environmental objectives.

These contributions offer a robust and sustainable approach to optimizing transportation systems in the face of environmental uncertainty and align with global goals for greener logistics.

3 | Preliminaries and Definitions

The basic definitions of the Farmatean fuzzy programming, which are used in our proposed work, which is given below:

Definition 1. According to [3], Farmatean fuzzy sets: A Farmatean Fuzzy Sets (FFSs) can be represented as $\tilde{\mathcal{F}} = \{\langle \omega, \alpha_{\tilde{\mathcal{F}}}(\omega), \beta_{\tilde{\mathcal{F}}}(\omega), \omega \in X \rangle\}$, Where $\alpha_{\tilde{\mathcal{F}}}(\omega), X \to [0,1]$ is the degree of satisfaction, and $\beta_{\tilde{\mathcal{F}}}(\omega), X \to [0,1]$ is the degree of dissatisfaction, including the conditions.

$$0 \leq \alpha_{\tilde{\mathcal{F}}}(\omega)^3 + \beta_{\tilde{\mathcal{F}}}(\omega)^3 \leq 1 \text{ for all } \omega \in X.$$

For any FFSs $\tilde{\mathcal{F}}$ and $\omega \in X$, $\sigma_{\tilde{\mathcal{F}}}(\omega) = \sqrt[3]{1-(\alpha_{\tilde{\mathcal{F}}}(\omega))^3-(\beta_{\tilde{\mathcal{F}}}(\omega))^3}$ is identified as the degree of indeterminacy of $\omega \in X$ to $\tilde{\mathcal{F}}$. The set $\tilde{\mathcal{F}} = \{\langle \omega, \alpha_{\tilde{\mathcal{F}}}(\omega), \beta_{\tilde{\mathcal{F}}}(\omega), \omega \in X \rangle\}$ is denoted as $\tilde{\mathcal{F}} = \langle \alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \rangle$."

Definition 2. Let $\tilde{\mathcal{F}} = \langle \alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \rangle$, $\tilde{\mathcal{F}}_1 = \langle \alpha_{\tilde{\mathcal{F}}_1}, \beta_{\tilde{\mathcal{F}}_1} \rangle$, and $\tilde{\mathcal{F}}_2 = \langle \alpha_{\tilde{\mathcal{F}}_2}, \beta_{\tilde{\mathcal{F}}_2} \rangle$ be three FFSs on the universal set X, and $\zeta > 0$ be any scalar; then the arithmetic operations of FFSs are as follows, with numerical examples.

$$\tilde{\mathcal{F}}_1 \oplus \tilde{\mathcal{F}}_2 = \left(\sqrt[3]{\alpha_{\tilde{\mathcal{F}}_1}{}^3 + \alpha_{\tilde{\mathcal{F}}_2}{}^3 - \alpha_{\tilde{\mathcal{F}}_1}{}^3\alpha_{\tilde{\mathcal{F}}_2}{}^3},\ \beta_{\tilde{\mathcal{F}}_1}\beta_{\tilde{\mathcal{F}}_2}\right).3.21. \quad (1)$$

Let $\tilde{\mathcal{F}} = \langle 0.4, 0.7 \rangle$, $\tilde{\mathcal{F}}_1 = \langle 0.8, 0.6 \rangle$ and $\tilde{\mathcal{F}}_2 = \langle 0.2, 0.9 \rangle$ be three FFSs and $\zeta = 2$ be any scalar quantity. Then,

$$\tilde{\mathcal{F}}_1 \oplus \tilde{\mathcal{F}}_2 = \langle 0.8, 0.6 \rangle \oplus \langle 0.2, 0.9 \rangle = (0.8020, 0.54),$$

$$\tilde{\mathcal{F}}_1 \otimes \tilde{\mathcal{F}}_2 = \left(\alpha_{\tilde{\mathcal{F}}_1}\alpha_{\tilde{\mathcal{F}}_2}, \sqrt[3]{{\beta_{\tilde{\mathcal{F}}_1}}^3 + {\beta_{\tilde{\mathcal{F}}_2}}^3 - {\beta_{\tilde{\mathcal{F}}_1}}^3{\beta_{\tilde{\mathcal{F}}_2}}^3}\right), \quad (2)$$

$$\tilde{\mathcal{F}}_1 \otimes \tilde{\mathcal{F}}_2 = \langle 0.8, 0.6\rangle \oplus \langle 0.2, 0.9\rangle = (0.16, 0.923).$$

$$\zeta \odot \tilde{\mathcal{F}} = \left(\sqrt[3]{1-(1-{\alpha_{\tilde{\mathcal{F}}}}^3)^{\zeta}}, {\beta_{\tilde{\mathcal{F}}}}^{\zeta}\right), \quad (3)$$

$$\zeta \odot \tilde{\mathcal{F}} = 2 \odot \langle 0.4, 0.7\rangle = (0.498, 0.49).$$

$$\tilde{\mathcal{F}}^{\zeta} = \left({\alpha_{\tilde{\mathcal{F}}}}^{\zeta}, \sqrt[3]{1-\left(1-{\beta_{\tilde{\mathcal{F}}}}^3\right)^{\zeta}}\right), \quad (4)$$

$$\tilde{\mathcal{F}}^{\zeta} = \langle 0.4, 0.7\rangle^2 = (0.064, 0.828).$$

Definition 3. "Let $\tilde{\mathcal{F}} = \langle\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}\rangle$, $\tilde{\mathcal{F}}_1 = \langle\alpha_{\tilde{\mathcal{F}}_1}, \beta_{\tilde{\mathcal{F}}_1}\rangle$, and $\tilde{\mathcal{F}}_2 = \langle\alpha_{\tilde{\mathcal{F}}_2}, \beta_{\tilde{\mathcal{F}}_2}\rangle$ be three FFSs on the universal set X, and $\zeta > 0$ be any scalar; then their arithmetic operations of FFS are defined as follows:

$$\tilde{\mathcal{F}}_1 \cup \tilde{\mathcal{F}}_2 = \left(\max\{\alpha_{\tilde{\mathcal{F}}_1}, \alpha_{\tilde{\mathcal{F}}_2}\}, \min\{\beta_{\tilde{\mathcal{F}}_1}, \beta_{\tilde{\mathcal{F}}_2}\}\right), \quad (5)$$

$$\tilde{\mathcal{F}}_1 \cup \tilde{\mathcal{F}}_2 = (\max\{\langle 0.8, 0.6\rangle\}, \min\{\langle 0.2, 0.9\rangle\}) = (0.8, 0.2).$$

$$\tilde{\mathcal{F}}_1 \cap \tilde{\mathcal{F}}_2 = \left(\min\{\alpha_{\tilde{\mathcal{F}}_1}, \alpha_{\tilde{\mathcal{F}}_2}\}, \max\{\beta_{\tilde{\mathcal{F}}_1}, \beta_{\tilde{\mathcal{F}}_2}\}\right), \quad (6)$$

$$\tilde{\mathcal{F}}_1 \cap \tilde{\mathcal{F}}_2 = (\min\{\langle 0.8, 0.6\rangle\}, \max\{\langle 0.2, 0.9\rangle\}) = (0.2, 0.6).$$

$$\tilde{\mathcal{F}}^c = (\beta_{\tilde{\mathcal{F}}}, \alpha_{\tilde{\mathcal{F}}}), \quad (7)$$

$$\tilde{\mathcal{F}}^c = \langle 0.4, 0.7\rangle^c = (0.7, 0.4).$$

Accuracy function of fermatean fuzzy sets

Suppose $\tilde{\mathcal{F}} = \langle\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}\rangle$ be an FFS, then the accuracy function of FFSs is represented as follows:

$$A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}) = \left({\alpha_{\tilde{\mathcal{F}}}}^3 + {\beta_{\tilde{\mathcal{F}}}}^3\right).$$

Theorem 1. Let $\tilde{\mathcal{F}}$ be an FFS $\tilde{\mathcal{F}} = \langle\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}\rangle$, then the score function $\tilde{\mathcal{F}}$ represented simply proceeds:

$$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}) = \frac{1}{2}\left(1 + {\alpha_{\tilde{\mathcal{F}}}}^3 - {\beta_{\tilde{\mathcal{F}}}}^3\right).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}})).$$

Property 1. Consider an FFS $\tilde{\mathcal{F}} = \langle\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}\rangle$, then $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}) \in [0,1]$.

Proof: According to the ortho-pair definition, $\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \in [0,1]$. Then, $\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}) \in [0,1]$, and also ${\alpha_{\tilde{\mathcal{F}}}}^3 \geq 0$, ${\beta_{\tilde{\mathcal{F}}}}^3 \geq 0$, ${\alpha_{\tilde{\mathcal{F}}}}^3 \leq 1$, and

$${\beta_{\tilde{\mathcal{F}}}}^3 \leq 1 \Rightarrow 1 - {\beta_{\tilde{\mathcal{F}}}}^3 \geq 0, \Rightarrow 1 + {\alpha_{\tilde{\mathcal{F}}}}^3 - {\beta_{\tilde{\mathcal{F}}}}^3 \geq 0, \therefore \frac{1}{2}\left(1 + {\alpha_{\tilde{\mathcal{F}}}}^3 - {\beta_{\tilde{\mathcal{F}}}}^3\right).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}})) \geq 0.$$

Again ${\alpha_{\tilde{\mathcal{F}}}}^3 - {\beta_{\tilde{\mathcal{F}}}}^3 \leq 1$, add one on both sides

$$\Rightarrow 1 + {\alpha_{\tilde{\mathcal{F}}}}^3 - {\beta_{\tilde{\mathcal{F}}}}^3 \leq 2 \ (\because {\alpha_{\tilde{\mathcal{F}}}}^3 \geq 0),$$

$$\Rightarrow \frac{1}{2}\left(1 + {\alpha_{\tilde{\mathcal{F}}}}^3 - {\beta_{\tilde{\mathcal{F}}}}^3\right).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}})) \leq 1 \ (\because \min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}) \leq 1).$$

Hence, $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}) \in [0,1]$.

Theorem 2. Let $\tilde{\mathcal{F}}$ be an FFS $\tilde{\mathcal{F}} = \langle\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}\rangle$, then the NFFSF $\tilde{\mathcal{F}}_{1d}$ represented simply as follows:

$$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{1d}) = \frac{1}{2}(1 + \alpha_{\tilde{\mathcal{F}}} - \beta_{\tilde{\mathcal{F}}}).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}))^2.$$

Property 2. Consider an FFS $\tilde{\mathcal{F}} = \langle\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}\rangle$, then $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{1d}) \in [0,1]$.

Proof: According to the ortho-pair definition, $\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \in [0,1]$. Then, $\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}) \in [0,1]$, and also $\alpha_{\tilde{\mathcal{F}}} \geq 0, \beta_{\tilde{\mathcal{F}}} \geq 0, \alpha_{\tilde{\mathcal{F}}} \leq 1$, and $\beta_{\tilde{\mathcal{F}}} \leq 1 \Rightarrow$

$$1 - \beta_{\tilde{\mathcal{F}}} \geq 0, \Rightarrow 1 + \alpha_{\tilde{\mathcal{F}}} - \beta_{\tilde{\mathcal{F}}} \geq 0, \therefore \frac{1}{2}(1 + \alpha_{\tilde{\mathcal{F}}} - \beta_{\tilde{\mathcal{F}}}).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}))^2 \geq 0.$$

Again, $\alpha_{\tilde{\mathcal{F}}} \leq 1$, and $\beta_{\tilde{\mathcal{F}}} \leq 1$, $\alpha_{\tilde{\mathcal{F}}} - \beta_{\tilde{\mathcal{F}}} \leq 1$, add one to both sides.

$$\Rightarrow 1 + \alpha_{\tilde{\mathcal{F}}} - \beta_{\tilde{\mathcal{F}}} \leq 2 \Rightarrow (\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}) \leq 1) \Rightarrow (\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}))^2 \leq 1,$$

$$\Rightarrow \frac{1}{2}(1 + \alpha_{\tilde{\mathcal{F}}} - \beta_{\tilde{\mathcal{F}}}).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}))^2 \leq 1 \ (\because (\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}}))^2 \leq 1).$$

Hence, $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{1d}) \in [0,1]$.

Theorem 3. Let $\tilde{\mathcal{F}}$ be an FFS $\tilde{\mathcal{F}} = \langle \alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \rangle$, then the Type 1 score function $\tilde{\mathcal{F}}_1$ represented as follows:

Type 1: Fermatean fuzzy score function $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{11}) = \frac{1}{2}(1 + \alpha_{\tilde{\mathcal{F}}}{}^2 - \beta_{\tilde{\mathcal{F}}}{}^2)$. According to the ortho-pair definition, $\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \in [0,1]$, and $\alpha_{\tilde{\mathcal{F}}}{}^2 \geq 0$, $\beta_{\tilde{\mathcal{F}}}{}^2 \geq 0$, $\alpha_{\tilde{\mathcal{F}}}{}^2 \leq 1$, and $\beta_{\tilde{\mathcal{F}}}{}^2 \leq 1$.

$$\Rightarrow 1 - \beta_{\tilde{\mathcal{F}}}{}^2 \geq 0, \Rightarrow 1 + \alpha_{\tilde{\mathcal{F}}}{}^2 - \beta_{\tilde{\mathcal{F}}}{}^2 \geq 0 \therefore \frac{1}{2}(1 + \alpha_{\tilde{\mathcal{F}}}{}^2 - \beta_{\tilde{\mathcal{F}}}{}^2) \geq 0.$$

Now, again, $\alpha_{\tilde{\mathcal{F}}}{}^2 - \beta_{\tilde{\mathcal{F}}}{}^2 \leq 1$, add on both sides.

$$\Rightarrow 1 + \alpha_{\tilde{\mathcal{F}}}{}^2 - \beta_{\tilde{\mathcal{F}}}{}^2 \geq 2 \ (\because \alpha_{\tilde{\mathcal{F}}}{}^2 \geq 0),$$

$$\Rightarrow \frac{1}{2}(1 + \alpha_{\tilde{\mathcal{F}}}{}^2 - \beta_{\tilde{\mathcal{F}}}{}^2) \geq (\because \langle \alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}} \rangle \leq 1).$$

Hence, $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{11}) \in [0,1]$. Similarly,

Type 2: Fermatean fuzzy score function $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{12}) = \frac{1}{3}(1 + 2\alpha_{\tilde{\mathcal{F}}}{}^3 - \beta_{\tilde{\mathcal{F}}}{}^3)$.

Type 3: Fermatean fuzzy score function $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_{13}) = \frac{1}{2}(1 + \alpha_{\tilde{\mathcal{F}}}{}^2 - \beta_{\tilde{\mathcal{F}}}{}^2).|\alpha_{\tilde{\mathcal{F}}} - \beta_{\tilde{\mathcal{F}}}|$.

"Let $\tilde{\mathcal{F}}_1 = \langle \alpha_{\tilde{\mathcal{F}}_1}, \beta_{\tilde{\mathcal{F}}_1} \rangle$, and $\tilde{\mathcal{F}}_2 = \langle \alpha_{\tilde{\mathcal{F}}_2}, \beta_{\tilde{\mathcal{F}}_2} \rangle$ be two FFSs, then the following operations will be satisfied:

$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) \geq S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_2)$ with $A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_1) > A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_2)$ if $\tilde{\mathcal{F}}_1 > \tilde{\mathcal{F}}_2$.

$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) \leq S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_2)$ with $A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_1) < A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_2)$ if $\tilde{\mathcal{F}}_1 < \tilde{\mathcal{F}}_2$.

$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) = S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_2)$ with $A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_1) = A_{\tilde{\mathcal{F}}}(\tilde{\mathcal{F}}_2)$ if $\tilde{\mathcal{F}}_1 = \tilde{\mathcal{F}}_2$."

Example 1. Let $\tilde{\mathcal{F}}_1 = \langle 0.7, 0.6 \rangle$ and $\tilde{\mathcal{F}}_2 = \langle 0.8, 0.5 \rangle$ be two FFSs, then we will see the following operations:

By using the score function

$$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}) = \frac{1}{2}(1 + \alpha_{\tilde{\mathcal{F}}}{}^3 - \beta_{\tilde{\mathcal{F}}}{}^3).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}})).$$

$$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) = \frac{1}{2}(1 + 0.7^3 - 0.6^3).(\min(0.7, 0.6)) = 0.337.$$

$$S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_2) = \frac{1}{2}(1 + 0.8^3 - 0.5^3).(\min(0.8, 0.5)) = 0.346.$$

Hence $S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) < S_{\tilde{\mathcal{F}}}{}^*(\tilde{\mathcal{F}}_1) \Rightarrow \tilde{\mathcal{F}}_1 < \tilde{\mathcal{F}}_2$.

Example 2. Let $\tilde{\mathcal{F}}_1 = \langle 0.9, 0.8 \rangle$ and $\tilde{\mathcal{F}}_2 = \langle 0.6, 0.5 \rangle$ be two FFS; then the following operations are represented. By using this score function,

$$S_{\tilde{\mathcal{F}}}^{*}(\tilde{\mathcal{F}}) = \frac{1}{2}\left(1 + \alpha_{\tilde{\mathcal{F}}}^{3} - \beta_{\tilde{\mathcal{F}}}^{3}\right).(\min(\alpha_{\tilde{\mathcal{F}}}, \beta_{\tilde{\mathcal{F}}})).$$

$$S_{\tilde{\mathcal{F}}}^{*}(\tilde{\mathcal{F}}_1) = \frac{1}{2}(1 + 0.9^3 - 0.8^3).(\min(0.9,0.8)) = 0.486.$$

$$S_{\tilde{\mathcal{F}}}^{*}(\tilde{\mathcal{F}}_2) = \frac{1}{2}(1 + 0.6^3 - 0.5^3).(\min(0.6,0.5)) = 0.022.$$

Hence $S_{\tilde{\mathcal{F}}}^{*}(\tilde{\mathcal{F}}_1) > S_{\tilde{\mathcal{F}}}^{*}(\tilde{\mathcal{F}}_1) \Rightarrow \tilde{\mathcal{F}}_1 > \tilde{\mathcal{F}}_2$. In the initial part of this section, the notations are outlined and presented in *Table 1*.

Table 1. Notations of the mathematical model.

i	Index for sources for all $i = 1,2,\ldots,m$.
j	index for destinations for all $j = 1,2,\ldots,n$.
m	Total number of sources.
n	Total number of destinations.
k	The number of conveyances, for all $k = 1,2,\ldots,K$.
t	The objective functions, for all $t = 1,2,\ldots,T$.
s_i	Supply capacity at source i.
d_j	Demand requirement at destination j.
e_k	Product shipment capacities of conveyance k.
x_{ij}	The quantity of goods transported from source i to destination j.
x_{ijk}	Number of goods transported by conveyance k from source i to destination j.
c_{ij}	Cost coefficient associated with transporting one unit from source i to destination j.
$C^{(t)}{}_{ijk}$	The unit transportation cost for the objective function at level t from the i^{th} source to the j^{th} destination via the k^{th} mode of conveyance.

3.1 | Mathematical Model

The formulation of the mathematical model of a traditional TP is defined as follows:

$$\begin{aligned} &\text{Min } f = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij},\\ &\text{s.t.}\\ &\sum_{j=1}^{n} x_{ij} \le s_i, (i = 1,2,\ldots,m),\\ &\sum_{i=1}^{m} x_{ij} \ge d_j, (j = 1,2.\ldots,n),\\ &x_{ij} \ge 0, \text{for all i and j.} \end{aligned} \tag{8}$$

Now, we use the FFP in this mathematical model under FFE. The mathematical model is presented as follows:

$$\begin{aligned} &\text{Minimize } f^{*} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{\tilde{\mathcal{F}}}\, x_{ij},\\ &\text{s.t.}\\ &\sum_{j=1}^{n} x_{ij} \le s_i^{\tilde{\mathcal{F}}}, (i = 1,2,\ldots,m),\\ &\sum_{i=1}^{m} x_{ij} \ge d_j^{\tilde{\mathcal{F}}}, (j = 1,2.\ldots,n). \end{aligned} \tag{9}$$

Such that

$s_i^{\tilde{\mathcal{F}}} = \left(\alpha_{\tilde{s}_i}, \beta_{\tilde{s}_i}\right)$, where $0 \le {\alpha_{\tilde{s}_i}}^3 + {\beta_{\tilde{s}_i}}^3 \le 1$,

$d_j^{\tilde{\mathcal{F}}} = \left(\alpha_{\tilde{d}_j}, \beta_{\tilde{d}_j}\right)$, where $0 \le {\alpha_{\tilde{d}_j}}^3 + {\beta_{\tilde{d}_j}}^3 \le 1$,

$C_{ij}^{\tilde{\mathcal{F}}} = \left(\alpha_{\tilde{c}_{ij}}, \beta_{\tilde{c}_{ij}}\right)$, where $0 \le {\alpha_{\tilde{c}_{ij}}}^3 + {\beta_{\tilde{c}_{ij}}}^3 \le 1$,

$x_{ij} \ge 0$, for all i and j.

Now, we convert this mathematical model into crisp form using NFFSF under FFEs.

$$\text{Minimize } f^* = \sum_{i=1}^{m}\sum_{j=1}^{n} S(C_{ij}^{\tilde{\mathcal{F}}})\, x_{ij},$$

$$\text{s.t.}$$

$$\sum_{j=1}^{n} x_{ij} \le S(s_i^{\tilde{\mathcal{F}}}), (i = 1,2,\ldots,m), \tag{10}$$

$$\sum_{i=1}^{m} x_{ij} \ge S(d_j^{\tilde{\mathcal{F}}}), (j = 1,2.\ldots,n),$$

$$x_{ij} \ge 0, \text{for all i and j.}$$

3.2 | Multi-Objective Transportation Problem Mathematical Model

The formulation for the MOTP mathematical model with FFP under the FFE is as follows:

$$\text{Minimize } f_t^* = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ijt}^{\tilde{\mathcal{F}}}\, x_{ij}, \text{ for all } t = 1, 2, \ldots, T,$$

$$\text{s.t.}$$

$$\sum_{j=1}^{n} x_{ij} \le s_i^{\tilde{\mathcal{F}}}, (i = 1,2,\ldots,m), \tag{10}$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j^{\tilde{\mathcal{F}}}, (j = 1,2.\ldots,n),$$

Such that

$s_i^{\tilde{\mathcal{F}}} = \left(\alpha_{\tilde{s}_i}, \beta_{\tilde{s}_i}\right)$, where $0 \le {\alpha_{\tilde{s}_i}}^3 + {\beta_{\tilde{s}_i}}^3 \le 1$,

$d_j^{\tilde{\mathcal{F}}} = \left(\alpha_{\tilde{d}_j}, \beta_{\tilde{d}_j}\right)$, where $0 \le {\alpha_{\tilde{d}_j}}^3 + {\beta_{\tilde{d}_j}}^3 \le 1$,

$C_{ij}^{\tilde{\mathcal{F}}} = \left(\alpha_{\tilde{c}_{ij}}, \beta_{\tilde{c}_{ij}}\right)$, where $0 \le {\alpha_{\tilde{c}_{ij}}}^3 + {\beta_{\tilde{c}_{ij}}}^3 \le 1$,

$x_{ij} \ge 0$, for all i and j,

Where $s_i^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{s}_i}, \beta_{\tilde{s}_i})$ units are available at the i^{th} supply node, and $d_j^{\tilde{\mathcal{F}}} = (\alpha_{\tilde{d}_j}, \beta_{\tilde{d}_j})$ units are in demand on the j^{th} demand node. Let the transportation cost $C_{ij}^{\tilde{\mathcal{F}}} = \left(\alpha_{\tilde{c}_{ij}}, \beta_{\tilde{c}_{ij}}\right)$ is the unit Fermatean fuzzy transportation cost and the i^{th} source node to the j^{th} demand node, and δ_{ij} is the number of items that are carried from the i^{th} source node to the j^{th} demand node. Now, we convert the MOTP mathematical model with FFP into a deterministic form using the proposed NFFSF. The crisp mathematical model of MOTP can be represented as follows:

$$\text{Minimize } f_t^* = \sum_{i=1}^{m}\sum_{j=1}^{n} S(C_{ijt}^{\tilde{\mathcal{F}}})\, x_{ij}, \quad t = 1, 2, \ldots, T,$$
$$\text{s.t.}$$
$$\sum_{j=1}^{n} x_{ij} \le S(s_i^{\tilde{\mathcal{F}}}), (i = 1,2,\ldots,m),$$
$$\sum_{i=1}^{m} x_{ij} \ge S(d_j^{\tilde{\mathcal{F}}}), (j = 1,2\ldots,n), \tag{11}$$
$$x_{ij} \ge 0, \text{for all i and j.}$$

4 | Methodology

The FFSs introduced by Senapati and Yager [3] represent a significant extension of intuitionistic fuzzy sets and offer enhanced flexibility in modeling uncertain environments. In FFSs, the sum of the truth and falsity membership degrees may exceed one, provided that the sum of their squares remains less than or equal to one. This unique property allows for a richer representation of uncertainty than intuitionistic and Pythagorean fuzzy sets. The authors also proposed fundamental operations and a score function to rank FFSs, laying the groundwork for applications in complex decision-making scenarios. Silambarasan [18] further contributed by examining the algebraic and operational properties of FFSs, thus strengthening the theoretical foundation of Fermatean fuzzy logic.

Building upon this foundation, Akram et al. [7] introduced interval-valued FFSs, offering an advanced mechanism to model incomplete and imprecise information. Their approach bypassed the need to convert fuzzy models into crisp equivalents, thereby preserving the richness of the fuzzy data and improving computational efficiency. This technique was applied to fractional TPs, demonstrating its effectiveness in fluctuating supply, demand, and cost parameters. In the broader context of fuzzy optimization, Zimmermann [19] pioneered the application of fuzzy linear programming to multi-objective problems, emphasizing the importance of compromise solutions when objectives conflict. His work laid the groundwork for modern fuzzy programming techniques that use linear, exponential, or hyperbolic membership functions to balance multiple goals. Over time, similar methodologies have been adapted to intuitionistic and Pythagorean fuzzy environments. However, these frameworks are often limited in their ability to model deep uncertainty, motivating the need for more robust tools like Fermatean fuzzy logic.

To address these challenges, we propose an FFPA explicitly designed to solve multi-objective decision problems in an FFE. This nonlinear programming framework simultaneously considers all objectives and accommodates uncertainty using FFP. The proposed model for fermatean fuzzy programming incorporates upper bounds U_t and lower bounds L_t for the objective function $f_t^*(x)$. Additionally, it involves the membership function $\mu(f_t^*(x))$ and non-membership function $\theta(f_t^*(x))$ for the objective function $f_t^*(x)$. This model aims to optimize decision-making under uncertainty, leveraging FFS to handle imprecision and uncertainty in objective functions. Including upper and lower bounds and membership and non-membership functions allows for a comprehensive representation of uncertainty, enabling robust decision-making in scenarios where precise information is lacking. Then, the proposed mathematical model for FFPA is as follows:

$$\text{Max } \delta\tau_1^3 - \tau_2^3,$$

where

$$\mu(f_t^*(x))^3 \ge \tau_1^3, \text{for all t},$$

$\theta\left(f_t^*(x)\right)^3 \leq \tau_2^3$, for all t.

The membership and non-membership function of objective functions are represented as follows:

$$\mu\left(f_t^*(x)\right) = \begin{cases} 1, & \text{if } f_t^*(x) \leq L_t, \\ \dfrac{U_t - f_t^*(x)}{U_t - L_t}, & \text{if } L_t \leq f_t^*(\delta) \leq U_t, \\ 0, & \text{if } f_t^*(x) \geq U_t. \end{cases} \tag{11}$$

And

$$\theta\left(f_t^*(x)\right) = \begin{cases} 0, & \text{if } f_t^*(x) \leq L_t, \\ \dfrac{f_t^*(x) - L_t}{U_t - L_t}, & \text{if } L_t \leq f_t^*(\delta) \leq U_t, \\ 1, & \text{if } f_t^*(x) \geq U_t. \end{cases}$$

i.e.,

$$(U_t - f_t^*(x))^3 \geq {d_t}^3{\tau_1}^3,\ (f_t^*(x) - L_t)^3 \leq {d_t}^3{\tau_2}^3,$$

where

$$d_t = U_t - L_t,$$

s.t.

$$x_{11} + x_{12} + \ldots\ldots\ldots\ldots + x_{1n} \leq s_1,$$

$$x_{21} + x_{22} + \ldots\ldots\ldots\ldots + x_{2n} \leq s_2,$$

: : : : : : :

: : : : : : :

$$x_{m1} + x_{m2} + \ldots\ldots\ldots\ldots + x_{mn} \leq s_m,$$

$$x_{11} + x_{21} + \ldots\ldots\ldots\ldots + x_{m1} \leq d_1,$$

$$x_{12} + x_{22} + \ldots\ldots\ldots\ldots + x_{m2} \leq d_2,$$

: : : : : : :

: : : : : : :

$$x_{1m} + x_{2m} + \ldots\ldots\ldots\ldots + x_{nm} \leq d_m,$$

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j, x_{ij} \geq 0, 0 \leq {\tau_1}^3, {\tau_2}^3 \leq 1, \leq {\tau_1}^3 + {\tau_2}^3 \leq 1, {\tau_1}^3 \geq {\tau_2}^3.$$

This formulation ensures the solution reflects the best compromise across all objectives, balancing cost-efficiency, timeliness, and environmental considerations within a single fuzzy decision-making framework. The FFPA is particularly suitable for real-world green TPs, where uncertainty is pervasive, and decision-makers must evaluate multiple sustainability-oriented goals concurrently.

4.1 | Solution Procedure

To effectively address the MOTP within the FFE, we propose a structured and comprehensive solution methodology based on the FFPA. This methodology is designed to handle multi-objective optimization's inherent uncertainty and complexity by integrating fuzzy logic with mathematical programming. The goal is to derive robust, compromise-based solutions that balance cost efficiency, environmental sustainability, and operational efficiency. The procedure begins by formulating the MOTP model in a Fermatean fuzzy framework, where all input parameters—such as transportation costs, supply capacities, demand requirements, and environmental indicators—are expressed as Fermatean fuzzy numbers. This formulation effectively captures uncertainty and imprecision in real-world logistics data.

Once the fuzzy model is established, it is converted into a deterministic (Crisp) form using the NFFSF. This transformation allows for the mathematical tractability of the fuzzy parameters, enabling the application of standard optimization solvers. Each objective function—typically including total transportation cost, total travel time, and carbon emissions—is then solved individually, allowing for extracting primary solutions and a better understanding of each objective's optimal behavior in isolation. Subsequently, a payoff matrix is constructed to evaluate the trade-offs among the objectives. This matrix captures the performance of each solution across all defined criteria and is used to determine the upper and lower bounds for each objective function. These bounds are computed using Fermatean fuzzy aggregation techniques, ensuring consistency with the fuzzy environment while preparing the model for compromise optimization.

The fully defined crisp model is reformulated in the final step using the proposed FFPA. This version integrates the calculated bounds and applies membership and non-membership functions to model satisfaction and dissatisfaction levels for each objective. The resulting FFPA model is then solved using the SciPy library in Python, a robust numerical computation and optimization tool. Through this approach, we obtain a compromise optimal solution that reflects balanced performance across all objectives under uncertainty. The implementation of this methodology is supported by a numerical example, with corresponding computations and results presented in *Tables 1–5*.

5 | Numerical

To demonstrate the applicability and effectiveness of the proposed methodology, we present a numerical example based on an MOTP under an FFE. In this problem, all parameters—including transportation cost, transportation time, carbon emissions, supply, and demand—are represented using FFPs. The objective is to optimize the transportation system simultaneously across three criteria: Minimizing total transportation cost, time, and carbon emissions, which are essential goals in green and sustainable logistics planning. The transportation network considered in this example consists of three sources and four destinations. The following tables contain the Fermatean fuzzy data for transportation cost, time, carbon emissions, and the supply-demand values for each node.

Table 1. Total transportation cost.

Source	β_1	β_2	β_3	β_4
α_1	(0.8, 0.7)	(0.7, 0.2)	(0.1, 0.6)	(0.2, 0.9)
α_2	(0.5, 0.8)	(0.1, 0.9)	(0.2, 0.6)	(0.2, 0.1)
α_3	(0.3, 0.4)	(0.7, 0.99)	(0.1, 0.8)	(0.7, 0.9)

Table 2. Total transportation time.

Source	β_1	β_2	β_3	β_4
α_1	(0.4, 0.8)	(0.7, 0.5)	(0.2, 0.9)	(0.6, 0.9)
α_2	(0.7, 0.5)	(0.1, 0.99)	(0.6, 0.8)	(0.4, 0.7)
α_3	(0.6, 0.8)	(0.8, 0.6)	(0.5, 0.1)	(0.3, 0.9)

Table 3. Carbon emissions cost.

Source	β_1	β_2	β_3	β_4
α_1	(0.5, 0.7)	(0.6, 0.8)	(0.2, 0.7)	(0.8, 0.7)
α_2	(0.4, 0.5)	(0.1, 0.2)	(0.8, 0.1)	(0.4, 0.7)
α_3	(0.8, 0.4)	(0.6, 0.4)	(0.4, 0.9)	(0.5, 0.9)

Table 4. Supply.

i	α_1	α_2	α_3
$(\alpha_{\tilde{\mathcal{F}}_i}, \beta_{\tilde{\mathcal{F}}_i})$	(0.3, 0.5)	(0.4, 0.8)	(0.6, 0.4)

Table 5. Demand.

j	β_1	β_2	β_3	β_4
$(\alpha_{\tilde{\mathcal{F}}_j}, \beta_{\tilde{\mathcal{F}}_j})$	(0.4, 0.7)	(0.2, 0.5)	(0.6, 0.4)	(0.2, 0.5)

Now, convert the Fermatean fuzzy data into crisp form using NFFSF. The crisp data of the proposed problem are represented as follows:

Table 6. Total transportation cost.

Source	β_1	β_2	β_3	β_4
α_1	(0.2695)	(0.03)	(0.0025)	(0.006)
α_2	(0.0875)	(0.001)	(0.012)	(0.0055)
α_3	(0.0405)	(0.1739)	(0.0015)	(0.196)

Table 7. Total transportation time.

Source	β_1	β_2	β_3	β_4
α_1	(0.048)	(0.15)	(0.006)	(0.126)
α_2	(0.15)	(0.00055)	(0.144)	(0.056)
α_3	(0.144)	(0.216)	(0.007)	(0.018)

Table 8. Carbon emissions cost.

Source	β_1	β_2	β_3	β_4
α_1	(0.1)	(0.144)	(0.01)	(0.2695)
α_2	(0.072)	(0.0045)	(0.0085)	(0.056)
α_3	(0.112)	(0.096)	(0.04)	(0.075)

Table 9. Supply.

i	α_1	α_2	α_3
$\alpha_{\tilde{\mathcal{F}}_i}, \beta_{\tilde{\mathcal{F}}_i}$	(0.036)	(0.048)	(0.096)

Table 10. Demand.

j	β_1	β_2	β_3	β_4
$(\alpha_{\tilde{\mathcal{F}}_j}, \beta_{\tilde{\mathcal{F}}_j})$	(0.056)	(0.014)	(0.096)	(0.014)

Since $\sum_{i=1}^{m} S(s_i^{\tilde{\mathcal{F}}}) = \sum_{j=1}^{n} S(d_j^{\tilde{\mathcal{F}}}) = 0.18$, the problem is balancing MOTP. We then solved the three TPs and obtained the basic feasible or optimal solutions for each objective. For the first objective function (Total transportation cost):

$$f_1^*(x) = 0.2695x_{11} + 0.03x_{12} + 0.0225x_{13} + 0.006x_{14} + 0.0875x_{21} + 0.001x_{22} + 0.012x_{23} + 0.0055x_{24} + 0.0405x_{31} + 0.1739x_{32} + 0.0015x_{33} + 0.196x_{34},$$

s.t.

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 0.036, x_{21} + x_{22} + x_{23} + x_{24} \leq 0.048,$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 0.096, x_{11} + x_{21} + x_{31} \leq 0.056, x_{11} + x_{22} + x_{32} \leq 0.014,$$
$$x_{13} + x_{23} + x_{33} \leq 0.096, x_{14} + x_{24} + x_{34} \leq 0.014, \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j, x_{ij} \geq 0.$$

After solving this problem using the Scipy optimization library, we obtain the optimal solution as follows:

$f_1^*(x)$: 1.6563000993102234e-1, $x_{11} =$ 6.701462050627256e-14,

x_{12} = 5.447537900889634e-13, x_{13} = 6.540701226004457e-13,

x_{14} = 2.3433877894449986e-12, x_{21} = 1.6269086281778882e-13,

x_{22} = 2.8037183688672324e-12, x_{23} = 1.1917212705073402e-12,

x_{24} = 2.4672960645959706e-12, x_{31} = 3.4491566715606586e-13,

x_{32} = 8.160054271726014e-14, x_{33} = 8.95759410927497e-12,

x_{34} = 8.134841315142161e-14.

For the second objective function (Total transportation time):

$$f_2^*(x) = 0.048x_{11} + 0.15x_{12} + 0.006x_{13} + 0.126x_{14} + 0.15x_{21} + 0.00055x_{22} + 0.144x_{23} + 0.056x_{24} + 0.144x_{31} + 0.216x_{32} + 0.007x_{33} + 0.018x_{34},$$

s.t.

$x_{11} + x_{12} + x_{13} + x_{14} \leq 0.036, x_{21} + x_{22} + x_{23} + x_{24} \leq 0.048,$

$x_{31} + x_{32} + x_{33} + x_{34} \leq 0.096, x_{11} + x_{21} + x_{31} \leq 0.056, x_{11} + x_{22} + x_{32} \leq 0.014,$

$x_{13} + x_{23} + x_{33} \leq 0.096, x_{14} + x_{24} + x_{34} \leq 0.014, \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j, x_{ij} \geq 0.$

After solving this problem using the Scipy optimization library, we obtain the optimal solution as follows:

$f_2^*(x)$: 4.211826137490883e-12, x_{11} = 8.263438891810004e-12,

x_{12} = 2.5838370001640527e-12, x_{13} = 6.335818003711425e-11,

x_{14} = 3.0146972331010592e-12, x_{21} = 2.5299779571885494e-12,

x_{22} = 8.790645598327858e-12, x_{23} = 2.669450392021306e-12,

x_{24} = 6.670724613805137e-12, x_{31} = 2.6400866542766385e-12,

x_{32} =1.800596343315708e-12, x_{33} =5.36625998899776e-11,

x_{34} = 2.1142895443476957e-11.

For the third objective function (Carbon emissions):

$$f_3^*(x) = 0.1x_{11} + 0.144x_{12} + 0.01x_{13} + 0.2695x_{14} + 0.072x_{21} + 0.0045x_{22} + 0.0085x_{23} + 0.056x_{24} + 0.112x_{31} + 0.096x_{32} + 0.04x_{33} + 0.075x_{34},$$

s.t.

$x_{11} + x_{12} + x_{13} + x_{14} \leq 0.036, x_{21} + x_{22} + x_{23} + x_{24} \leq 0.048,$

$x_{31} + x_{32} + x_{33} + x_{34} \leq 0.096, x_{11} + x_{21} + x_{31} \leq 0.056, x_{11} + x_{22} + x_{32} \leq 0.014,$

$x_{13} + x_{23} + x_{33} \leq 0.096, x_{14} + x_{24} + x_{34} \leq 0.014, \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j, x_{ij} \geq 0.$

After solving this problem using the Scipy library, we obtain the optimal solution as follows:

f_3^*: 7.110957493321067e-12, $x_{11} = 6.1746254985444975e-12$,

x_{12} = 4.317295196854511e-12, x_{13} = 6.194365813830885e-11,

x_{14} = 2.30864320123096e-12, x_{21} = 8.587199756168197e-12,

x_{22} = 7.54519853118348e-11, x_{23} = 6.803357681972516e-11,

x_{24} =1.0962059083018053e-11, x_{31} = 5.5603385632255506e-12,
x_{32} =6.461011159919284e-12, x_{33} =1.5425548584548254e-1,

x_{34} =8.269047323003344e-12.

After obtaining the solutions for all objectives individually, we obtain the payoff matrix in *Table 11*.

Table 11. Payoff matrix.

	Δ_1	Δ_2	Δ_2
f_1^*	1.6563000993102234e-13 = (0.0000037438382)	0.000888743	0.001341743
f_2^*	0.0018528783	4.211826137490883e-12 = (0.0000258783)	0.00995687
f_3^*	0.0115596912	0.014590691	7.110957493321067e-12 = (0.00004369123)

So, we can find the upper and lower bounds for all objective functions and $d_t = U_t - L_t$, which are as follows:

$L_1 = 0.0000037438382$, $U_1 = 0.001341743$, $d_1 = 0.001337991618$,

$L_2 = 0.0000258783$, $U_2 = 0.00995687$, $d_2 = 0.0099309917$,

$L_3 = 0.00004369123$, $U_3 = 0.014590691$, $d_3 = 0.01454699977$.

Now, we solved the mathematical model using the proposed FFPA. Where

$$\mu\left(f_t^*(x)\right)^3 \geq \tau_1{}^3, \text{for all t,} \qquad \theta\left(f_t^*(x)\right)^3 \leq \tau_2{}^3, \text{for all t.}$$

i.e.,

$$(U_t - f_t^*(x))^3 \geq d_t{}^3\tau_1{}^3, \ (f_t^*(x) - L_t)^3 \leq d_t{}^3\tau_2{}^3,$$

Where

$d_t = U_t - L_t$.

Now, we calculate the lower and upper bounds of the proposed problem.

$$\Rightarrow (0.001341743 - f_1^*)^3 \geq 0.0000000023897975\tau_1{}^3,$$

$$\Rightarrow (0.00995687 - f_2^*)^3 \geq 0.000000979146657\tau_1{}^3,$$

$$\Rightarrow (0.014590691 - f_3^*)^3 \geq 0.000003077731643\tau_1{}^3,$$

$$\Rightarrow (f_1^* - 0.0000037438382)^3 \geq 0.0000000023897975\tau_2{}^3,$$

$$\Rightarrow (f_2^* - 0.0000258783)^3 \geq 0.000000979146657\tau_2{}^3,$$

$$\Rightarrow (f_3^* - 0.00004369123)^3 \geq 0.000003077731643\tau_2{}^3,$$

Subject to

$x_{11} + x_{12} + \ldots\ldots\ldots\ldots\ldots\ldots + x_{1n} \leq s_1$,

$x_{21} + x_{22} + \ldots\ldots\ldots\ldots\ldots\ldots + x_{2n} \leq s_2$,

: : : : : : :

$: \quad : \quad : \quad : \quad : \quad : \quad :$

$x_{m1} + x_{m2} + \ldots\ldots\ldots\ldots\ldots\ldots + x_{mn} \leq s_{m,}$

$x_{11} + x_{21} + \ldots\ldots\ldots\ldots\ldots\ldots + x_{m1} \leq d_1,$

$x_{12} + x_{22} + \ldots\ldots\ldots\ldots\ldots\ldots + x_{m2} \leq d_{2,}$

$: \quad : \quad : \quad : \quad : \quad : \quad :$

$: \quad : \quad : \quad : \quad : \quad : \quad :$

$x_{1m} + x_{2m} + \ldots\ldots\ldots\ldots\ldots\ldots + x_{nm} \leq d_m,$

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j, x_{ij} \geq 0, 0 \leq {\tau_1}^3, {\tau_2}^3 \leq 1, \leq {\tau_1}^3 + {\tau_2}^3 \leq 1, {\tau_1}^3 \geq {\tau_2}^3.$$

After solving the above problem using the Scipy optimization library, the optimal solution of the proposed problem is represented as follows:

${f_1}^* = 0.004191344499158313$, ${f_2}^* = 0.003977749589394637$,

${f_3}^* = 0.0020184923860561447$, $\tau_1 = 0.001289240882763909$,
$\tau_2 = 0.0006606981979325261$, $y_{11} = 0.30573288027448287$,

$y_{12} = 0.02845428401361542$, $y_{13} = 0.0011005037929001924$,

$y_{14} = 0.0010085155145783693$, $y_{21} = 0.017421950162707155$,

$y_{22} = 0.001010127111597806$, $y_{23} = 0.0011462979936144556$,

$y_{24} = 0.0010634351469773963$, $y_{31} = 0.01747142198034052$,

$y_{32} = 0.0010542754473920062$, $y_{33} = 0.02057583566682344$,

$y_{34} = 0.0009997035453720312$.

6 | Conclusion

In this study, we developed a robust and flexible mathematical framework to address green MOTP under uncertainty using the FFPA. We began by formulating the traditional TP. We extended it to an MOTP that minimizes transportation cost, time, and carbon emissions—three critical dimensions in sustainable logistics and environmental management. To effectively handle the vagueness inherent in real-world transportation data, we FFP and utilized the NFFSF to convert fuzzy data into crisp, solvable forms. The proposed FFPA approach demonstrates strong potential in delivering compromise optimal solutions for multi-objective decision-making scenarios, particularly in the context of green transportation systems. By incorporating environmental objectives into the optimization process, our method supports more informed and balanced decision-making, where economic efficiency does not come at the expense of ecological sustainability. The numerical example validated the practical applicability of the proposed model and highlighted its ability to navigate trade-offs between conflicting goals in uncertain environments. This work contributes a valuable tool for planners and decision-makers seeking to design eco-efficient transportation networks. The FFPA framework can be readily adapted to a range of real-world applications, and its compatibility with other fuzzy environments further enhances its relevance for future research in sustainable and resilient logistics.

Data Availability

Required data is available in this manuscript.

Conflict of Interest

There are no competing interests to declare.

Consent for Publication

All authors have provided their consent for the publication of this manuscript.

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