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Assessing Medical Waste Treatment Technique based on Hyperbolic fuzzy EM-SWARA with COPRAS and ARAS Approaches

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Abstract

Medical Waste Treatment Techniques (MWTT) have become a significant concern due to the imminent risks they pose to human health and the environment. Proper and secure treatment and disposal of toxic and harmful medical waste are essential and various MWTT options are available to achieve this. The selection of the ideal MWT option is a complex and crucial Multi Criteria Decision Making (MCDM) problem as the decision is influenced by several factors both qualitative and quantitative aspects. This study presents a hybrid MCDM method for analyzing and opting the MWT options within a Hyperbolic fuzzy framework. The Hyperbolic Fuzzy Set (HyFS) is an advanced tool that addresses uncertainty with greater precision, providing more flexibility for the decision makers. An entropy measure and a score function have been introduced in a hyperbolic fuzzy environment. Objective weights are evaluated using the entropy measure while subjective weights are assessed through the Stepwise Weight Assessment Ratio Analysis (SWARA) model. Consequently, a pioneering hybrid MCDM approach is presented combining HyF-EM-SWARA with Complex Proportional Assessment (COPRAS) and Additive Ratio Assessment (ARAS) techniques to identify the optimal MWT option in India. Furthermore, relative evaluations and variability analysis are presented to demonstrate the stability and reliability of the proposed hybrid MCDM methods for ranking the preferences of MWTT.

Keywords: Hyperbolic Fuzzy Set, Entropy Measure, Medical Waste Treatment.

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1|Introduction

In our everyday lives, we are constantly faced with the need to make decisions, guided by a variety of conditions and pieces of information. Yet, this information often carries a certain vagueness and uncertainty, making over choices all the more challenging and intriguing. To address such situation, Zadeh[1] introduced fuzzy set theory (FST). which has numerous applications in real world domains, including risk assessment, operation research, medical diagnosis, decision making etc. FST assigns a value within the interval [0,1] to each element in a universal set known as membership value/degree of satisfaction(α) while non-membership value/degree of dissatisfaction(β) as $1-\alpha$. But the value of non-membership may not be always one minus the membership value. Due to these limitations, Atanassov[2] introduced the Intuitionistic fuzzy set (IFS), as a generalisation of FST. This framework imposes the condition that the sum of membership value α and non-membership value β cannot exceed 1 i.e., $0 \leq \alpha+\beta \leq 1$ where $\alpha, \beta \in [0,1]$. Nonetheless, in specific scenarios, where $\alpha = 0.7$ and $\beta = 0.4$, this condition may not be met by IFS. Due to limitation of IFS, Yager[3] introduce the Pythagorean Fuzzy Set (PFS), a more adaptable framework. In the realm of PFS, the sum of the squares of membership and non-membership values is constrained to not exceed 1 i.e., $0 \leq \alpha^2+\beta^2 \leq 1$. However, it also has its own limitations. For instance, when one consider values such as $\alpha = 0.7$ and $\beta = 0.8$, we find that this condition cannot be satisfied. Later, Yager[4] extended the PFS to q-rung orthopair fuzzy set (Q-ROFS), where the q^{th} sum of membership and non-membership values is restricted to not exceed 1 i.e., $0 \leq \alpha^q+\beta^q \leq 1$ where $q \geq 1$. Q-ROFS significantly expands the decision-making framework, providing greater flexibility compared to other existing fuzzy sets.

In practical scenarios, we often encounter situation where individuals experience complete satisfaction alongside partial dissatisfaction. This duality allows for a possibility of full membership coupled with partial non-membership or vice versa. To address this complexity, Dutta and Borah[5] proposed the Hyperbolic Fuzzy Set (HyFS), which serves as generalisation of IFS, PFS and Q-ROFS. In HyFS, the product of membership and non-membership values cannot exceed 1 i.e., $0 \leq \alpha.\beta \leq 1$ thus providing significant flexibility and versatile framework for decision-makers, allowing for scenarios where $\alpha = 1$ and $\beta > 0$ or $\alpha > 0$ and $\beta = 1$, where $\alpha, \beta \in [0,1]$. None of the IFS, PFS and Q-ROFS can meet this nuanced requirement making HyFS as a significant advancement in the field of fuzzy sets.
In recent years, the global population has surged, bringing with it a dramatic rise in healthcare facilities and an ever-growing demand for medical services. Within this context, effective medical based treatment has become essential for protecting public health, preventing environmental pollution and ensuring compliance with regulatory standards. Healthcare facilities produce various types of waste, including infectious materials, hazardous substances and pharmaceuticals, all of which can pose serious risks if not managed properly. Ineffective waste management not only threatens our environment but also invites unpleasant odors and attracts pests, such as insects and rodents, which can facilitate the spread of diseases like bacterial infections, HIV and hepatitis. Therefore, selecting the appropriate medical waste treatment technology in crucial for ensuring safety, regulatory compliance, environmental sustainability and cost-effectiveness. The decision-making process in this area is complex and influenced by several factors that can effectively analysed using Multi-Criteria Decision-Making (MCDM) methods. To address this complexity, we propose an integrated framework that evaluates conflicting criteria based on expert assessments. Remarkably, no prior research has explored the application of hybrid MCDM methods within a hyperbolic fuzzy framework, making our approach innovative. Thus, we present the optimal medical waste treatment techniques identified through the HyF-EM-SWARA-COPRAS and HyF-EM-SWARA-ARAS approaches. This research aims to contribute to the development of improved strategies for managing medical waste, ensuring a safer and more sustainable environment.

1.1|Literature Review

Zadeh[1] introduced the concept of fuzzy set theory to address the issue of vagueness, uncertainty and imprecision inherent in real-life decision-making scenarios. Since then, numerous extensions of fuzzy sets have emerged, including IFS[2], PFS[3] and Q-ROFS[4], each serving as a generalization of traditional fuzzy sets. More recently, Dutta and Borah[5] have developed the HyFS which offers significant flexibility and enough space to the diverse need of decision-makers. The HyFS has garnered substantial interest among researchers focused on decision-making amidst the increasing complexity of contemporary challenges. Numerous studies have utilized

HyFS to tackle a variety of MCDM problems. Divsalar et al[6] effectively applied HyFS to the sustainable supplier selection problem within the daily industry. Zavadskas et al[7] employed hyperbolic fuzzy data to establish personalized priorities for contract clauses. Additionally, Banik and Dutta[8] employed HyFS to identify crime-prone zones in Dibrugarh city.

The selection of optimal MWTT is crucial for safeguarding public health and protecting the environment. Extensive research has been conducted to identify the most effective strategies for enhancing MWTT, with fuzzy set theory being a key component of these studies. However, there is currently no published evaluations concerning MWTT assessment using the HyFS preference structure. The MCDM methods established in fuzzy environment have demonstrated their efficacy in the selection process for MWTT, highlighting the necessity for further exploration and application of HyFS in this essential area. To address the significant challenges associated with MWTT through traditional decision-making methods[9, 10]. Dursun et al[11] have pioneered a MCDM technique within a fuzzy framework for MWT. Their work introduced two robust fuzzy MCDM frameworks, employing fuzzy integral and a hierarchical distance-based technique which represent a sustainable advancement in this field. Building on Dursun et al[11] insights, Liu et al[12] advanced the determining the optimal selection for MWT. This was followed by the development of an ILT-MULTIOORA technique by Liu et al[13] which utilizes interval 2-tuple linguistic variables. Liu et al[14] evaluated the weights of the criteria using the 2-tuple DAMETAL method and applied the fuzzy MULTIMOORA methodology to assess MWTT comprehensively. Lu et al[15] introduced an interval 2-tuple induced-TOPSIS method and Xiao[16] unvieled a technique for prioritizing MWTT that assigns crisp values to linguistic variables and leverages D-numbers integration for comprehensive assessment. Hinduja and Pandey[17] formulated a hybrid algorithm that enhances the selection for MWTT. Additionally, Li et al[18] contributed to the dialogue by proposing a hybrid MCDM technique that synergizes the DEMATEL and TOPSIS methods within an interval-valued fuzzy environment. Despite these advancements, a notable gap persists in the literature regarding methodologies that seamlessly integrate both objective and subjective weights for MWT attributes. Both weights plays a vital role in minimizing potential losses while enhancing decision accuracy. The MWT selection problem remains largely unexplored. There exists a significant gap in research utilizing an integrated entropy measure, SWARA model combined with COPRAS or ARAS approaches in a hyperbolic fuzzy environment. Therefore, this research proposes an innovative approach to MWTT selection within a hyperbolic fuzzy environment, effective harnessing the strengths of both objective and subjective weights.

1.2|Motivations and Objectives

By employing MCDM, healthcare facility can adopt efficient solutions that not only comply with regulatory standards, but also advanced sustainability goals. This strategic approach is vital for safeguarding public health and protecting our environment for future generations. One constructive approach to enhancing contemporary MCDM techniques involves carefully establishing appropriate weights for various criteria.

In consideration of the preceding points, the following motivation drive this research :

1. Existing fuzzy set theories often encounter challenges in addressing certain paradoxical situations, creating a significant gap in the handling of uncertainty. The implementation of hyperbolic fuzzy set offers a sophisticated approach that allows for full membership, partial non-membership and their reversals.
2. Entropy measure and score functions plays a pivotal role in determining the weights of criteria and evaluate alternatives within MCDM models. These methodologies can facilitate the development of advanced MCDM techniques such as COPRAS, SAW, ARAS etc. Furthermore, there is no development of entropy measure within a hyperbolic fuzzy environment, highlighting a valuable opportunity for innovation in this area.
3. There is a significant lack of literature addressing the application of hybrid MCDM methods within the hyperbolic fuzzy context.

In consideration of the preceding points, the following are the objectives of this research :

1. Developing a novel score function for HyFS.
2. Developing an entropy measure for HyFS for determining the objective weight of the criteria.
3. A case study to endorse the probable impact of the proposed algorithm in MWTT selection using HyFS.

1.3|Structure of the paper

The structure of this paper is organized as follows :
In section 2, we provide the definitions of existing fuzzy sets, including IFS, PFS, Q-ROFS and HyFS with some basic operations on HyFS. Section 3 presents a novel entropy measure, score function within hyperbolic fuzzy environment and the algorithms for the EM-SWARA-COPRAS and EM-SWARA-ARAS methods under the HyFSs context. Section 4 demonstrates the practical application of the developed methodology through a case study on selecting the most desirable MWTT option in India. The case study illustrates the strength and robustness of the proposed approach. Also, a comparative analysis is presented that underscores its advantages. Finally, section 5 conclude the paper and outlines future research scope.

2|Preliminaries

This section summarizes some of the definitions and notations necessary for a comprehensive understanding of the study.

Definition 1.[4] Let, $X = \{x_1, x_2, ..., x_n\}$ represents a Universe of Discourse. A q-rung orthopair fuzzy set (Q-ROFS) Q on X is defined by $Q = \{(x_i, \alpha_Q(x_i), \beta_Q(x_i))|x_i \in X\}$ where $\alpha_Q\colon X \to [0,1]$ denotes the membership function and $\beta_Q\colon X \to [0,1]$ denotes the non-membership function of the element $x_i \in X$ with the restriction $0 \leq (\alpha_Q(x_i))^q + (\beta_Q(x_i))^q \leq 1$ where $q \geq 1$.
The degree of hesitancy is determined by $\pi_Q(x_i) = \{1 - \alpha_Q(x_i))^q - (\beta_Q(x_i))^q\}^{\frac{1}{q}}$.

Definition 2. A q-rung orthopair fuzzy set is classified as follows :
Intuitionistic Fuzzy Set (IFS)[2] if $q = 1$ i.e., having the condition $0 \leq (\alpha_Q(x_i)) + (\beta_Q(x_i)) \leq 1$
Pythagorean Fuzzy Set(PFS)[3] if $q = 2$ i.e., having the condition $0 \leq (\alpha_Q(x_i))^2 + (\beta_Q(x_i))^2 \leq 1$
Fermatean Fuzzy Set(FFS)[43] if $q = 3$ i.e., having the condition $0 \leq (\alpha_Q(x_i))^3 + (\beta_Q(x_i))^3 \leq 1$
Quartic Fuzzy Set(QrFS)[44] if $q = 4$ i.e., having the condition $0 \leq (\alpha_Q(x_i))^4 + (\beta_Q(x_i))^4 \leq 1$
Quintic Fuzzy Set(QnFS)[45] if $q = 5$ i.e., having the condition $0 \leq (\alpha_Q(x_i))^5 + (\beta_Q(x_i))^5 \leq 1$

Definition 3.[5] Let, $X = \{x_1, x_2, ..., x_n\}$ represents a Universe of Discourse. A Hyperbolic Fuzzy Set (HyFS) Λ on X is defined by $H = \{(x_i, \alpha_\Lambda(x_i), \beta_\Lambda(x_i))|x_i \in X\}$ where $\alpha_\Lambda\colon X \to [0,1]$ denotes the membership function and $\beta_\Lambda\colon X \to [0,1]$ denotes the non-membership function of the element $x_i \in X$ with the restriction $0 \leq (\alpha_\Lambda(x_i)).(\beta_\Lambda(x_i)) \leq 1$.

Definition 4. Let, $\Lambda_1 = (\alpha_1, \beta_1)$ and $\Lambda_2 = (\alpha_2, \beta_2)$ be two HyFSs with $\lambda > 0$. Then, the operations on HyFSs are defined as follows :
$(i)\Lambda_1 \cup \Lambda_2 = (max\{\alpha_1, \alpha_2\}, min\{\beta_1, \beta_2\})$
$(ii)\Lambda_1 \cap \Lambda_2 = (min\{\alpha_1, \alpha_2\}, max\{\beta_1, \beta_2\})$
$(iii)\Lambda_1^C = (1 - \alpha_1, 1 - \beta_1)$
$(iv)\Lambda_1 \bigoplus \Lambda_2 = (\alpha_1 + \alpha_2 - \alpha_1\alpha_2, \beta_1\beta_2)$
$(v)\Lambda_1 \bigotimes \Lambda_2 = (\alpha_1\alpha_2, \beta_1 + \beta_2 - \beta_1\beta_2)$
$(vi)\lambda\Lambda_1 = (1 - (1 - \alpha_1)^\lambda, \beta_1^\lambda)$
$(vii)\Lambda_1^\lambda = (\alpha_1^\lambda, 1 - (1 - \beta_1^\lambda))$
$(viii)\Lambda_1 \subseteq \Lambda_2$ iff $\alpha_1 \leq \alpha_2$ and $\beta_1 \geq \beta_2$

Definition 5. Let, $\Lambda_i = (\alpha_i, \beta_i), i = 1, 2, ..., n$ be HyFSs. Then, the hyperbolic fuzzy weighting averaging operator ($HyFWAO$) is defined as
$HyFWA_w(\alpha_1, \alpha_2, ..., \alpha_n) = w_j\alpha_j = \bigoplus_{j=1}^n (1 - \prod_{j=1}^n (1 - \alpha_j)w_j, \prod_{j=1}^n (\beta_j)^{w_j})$
where $w = (w_1, w_2, ..., w_n)^T$ is a weight vector for α_j and $\sum_{j=1}^n w_j = 1$ with $0 \leq w_j \leq 1, i = 1, 2, ..., n$.

3|Proposed work

This section presents a novel entropy measure and a score function within the hyperbolic fuzzy framework, making a significant advancement in MCDM techniques.

3.1|A novel hyperbolic fuzzy entropy measure

The entropy measure plays a pivotal role in the development of MCDM techniques, as it is essential for identifying the objective weight of attributes. Despite its importance, the hyperbolic fuzzy framework has lacked a suitable entropy measure until now.
Definition 5. Let, HyFN $H = (x, y)$ such that $x, y \in [0, 1]$ and $0 \leq x.y \leq 1$.
A function $E\colon HyFS(X) \to [0, 1]$ is a HyF entropy measure if
$(i)E(H) = 0$ if H is a crisp set.
$(ii)E(H)$ attains its unique maximum when $x = y = \frac{1}{2}$
$(iii)E(H) = E(H^C)$
$(iv)E(H_1) \leq E(H_2)$ if H_1 is crisper than H_2, i.e., $x_1 \leq x_2 \leq \frac{1}{2}, y_1 \leq y_2 \leq \frac{1}{2}$ or $x_1 \geq x_2 \geq \frac{1}{2}, y_1 \geq y_2 \geq \frac{1}{2}$ where $H_1 = (x_1, y_1)$ and $H_2 = (x_2, y_2)$
A novel HyF entropy measure is proposed as given below :

$$E_{NP}(H) = 1 - \frac{(2x + 2y - 2)}{xy + 1} \tag{1}$$

Theorem 1. The function $E_{NP}(\Lambda)$ is an HyF entropy measure.

3.2|A novel hyperbolic fuzzy score function

We propose a novel score function with the following definitions :
Definition 6. Let, $H = (x, y)$ be a $HyFN$, then the score function S_{NP} of H is defined as

$$S_{NP}(H) = \frac{xy + x + 1}{2(y + 1)} \tag{2}$$

where $S_{NP}(H) \in [0, 1]$

4|Application in MWTT selection

Medical waste treatment has experienced considerable advancement in recent years due to the growing recognition of public health and environmental imperatives. Various case studies in developing countries have been conducted to identify the optimal MWTT. This study aims to rigorously evaluate the available alternatives for MWT in order to determine the most effective option. The study outlines five prominent technique A_1: autoclaving, A_2: microwave, A_3: plasma pyrolysis, A_4: chemical disinfection, A_5: incineration for MWT in India, which are assessed against ten critical criteria. A team of four experts $\{E_1, E_2, E_3, E_4\}$ is assembled who provided valuable insights into the significance of each criteria[46, 20]. The linguistic variables matrix employed to evaluate the criteria is detailed in Table 1. The experts evaluations of LVs are summarized in Table 2. To effectively rank the MWTTs, the proposed HyF-EM-SWARA-COPRAS and HyF-EM-SWARA-ARAS algorithms are utilized. Moreover, this study is pioneering as it employs a hybrid MCDM method within a hyperbolic fuzzy framework for the selection of MWTT. This research seeks to make a meaningful contribution to the field of medical waste management and public health.

The weights of the DEs are calculated with the results presented in Table 3. For each alternative based on the criteria, a HyF decision matrix is developed by the panel of experts as illustrated in Table 4. The AHyF-DM is then constructed using DE's weight. The integrated weights for the criteria are determined by combining objective and subjective weights. The objective weights are derived using the Entropy Measure. The entropy values are calculated by employing equation(1) and Table 5. The objective weight is evaluated as follows :
$O_j = \{0.0924, 0.1141, 0.1315, 0.1046, 0.0903, 0.1248, 0.0905, 0.0686, 0.1135, 0.0697\}$

The SWARA model is employed to determine the subjective weights of the criteria. The subjective weight of the criteria are determined using SWARA model is presented in the last column of Table 7. Finally, the integrated/final weights of the criteria are determined. The final weights of the criteria are presented as follows :
$w_j = \{0.1094, 0.1195, 0.1275, 0.1121, 0.0977, 0.1043, 0.0865, 0.0749, 0.0956, 0.0725\}$

Initially, the COPRAS method is applied to assess the alternative for MWT methods in India. The

Table 1. LVs for the criteria based on HyFNs.

Importance	**HyFNs**
Extremely good (EG)	(1,0)
Very good (VG)	(0.95,0.05)
Good (G)	(0.75,0.20)
Medium good (MG)	(0.65,0.30)
Fair (F)	(0.55,0.40)
Medium poor (MP)	(0.40,0.50)
Poor (P)	(0.30,0.60)
Very poor (VP)	(0.20,0.70)
Extremely poor (EP)	(0.10,0.80)

Table 2. LVs for the level of significance of DEs.

Importance	**HyFNs**
Extremely significant	(1,0)
Very significant	(0.90,0.05)
Significant	(0.70,0.20)
Moderate	(0.60,0.30)
Insignificant	(0.40,0.50)
Very insignificant	(0.30,0.80)
Extremely insignificant	(0.10,0.80)

Table 3. HyF linguistic decision matrix by DEs.

Alternatives	Experts	Criteria									
		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	E_1	G	P	P	VG	F	F	F	VG	G	VG
	E_2	F	MG	F	G	MG	P	G	P	VP	G
	E_3	MP	G	P	P	G	P	VG	F	MG	VG
	E_4	F	F	P	P	P	VG	MG	F	G	MG
A_2	E_1	VG	VP	F	G	F	MP	VG	G	G	VG
	E_2	P	G	F	G	MG	P	G	F	MP	MG
	E_3	F	MG	P	MP	G	MG	G	MG	F	G
	E_4	F	MG	VP	P	MP	G	VG	F	VG	P
A_3	E_1	G	VP	F	G	F	MP	VG	G	G	VG
	E_2	P	G	G	G	P	F	F	F	P	VG
	E_3	F	G	P	P	G	MP	F	F	F	G
	E_4	P	VP	F	VP	MG	G	G	G	MG	MP
A_4	E_1	V	MP	MG	G	MG	MG	P	VG	MG	F
	E_2	P	MG	F	MG	MG	G	MG	G	P	MG
	E_3	F	F	P	P	VG	MP	F	G	F	MG
	E_4	F	P	VP	P	MP	G	G	VG	D	MP
A_5	E_1	VG	G	F	VG	F	MG	F	VG	MG	G
	E_2	VG	VG	MG	F	MG	G	MG	MG	G	G
	E_3	P	P	G	P	G	MP	F	G	P	MP
	E_4	P	P	MG	MG	MP	F	F	G	F	P

Table 4. Weights of the DEs.

DEs	E_1	E_2	E_3	E_4
Ratings	Significant	Moderate	Very significant	Insignificant
HyFNs	(0.70,0.20)	(0.60,0.30)	(0.90,0.05)	(0.40,0.50)
Weights (v_k)	0.2697	0.2259	0.3555	0.1489

relative grade and utility grade are presented in Table 8. The results in Table 8 distinctly indicates that the ranking order for MWTT is $A_1 > A_2 > A_4 > A_3 > A_5$. Alternative A_1 emerges as the best option for MWTT. Moreover, we strengthen our analysis by applying the ARAS method within hyperbolic fuzzy framework. Table 12 shows that the alternatives are ranked as $A_1 > A_2 > A_4 > A_3 > A_5$. Thus, A_1 is the best option for MWTT.

4.1|Comparative Analysis

In this section, we present comparison of our proposed technique against existing methodologies, demonstrating the efficacy and advantage of our developed approach for MWTT selection.

In comparison with other developed methodologies for MWTT selection, our approach demonstrates the following strengths :
1. Methodologies[12, 28, 46, 47, 49, 50] focuses solely on objective weights, disregarding subjective evaluations while methodologies[14, 16, 19, 22, 48, 51] emphasizes subjective weights, disregarding objective weights. This imbalance results in a flawed evaluation, allowing decision-makers to harness valuable human insight while maintaining rigorous analytical standards.
2. In methodologies[14, 16, 28, 47, 49, 50] weights assigned to experts are assumed randomly rather than meticulously computed, resulting in potentially biased and unrealistic outcomes. In critical fields like healthcare and medical waste management such oversights can have profound consequences. Our proposed methodology methodically computes expert weights, ensuring accuracy and reliability.
3. Methodologies[16, 20, 51] neglects the critical nature of attributes which can adversely affect the ranking of alternatives. Our proposed methodology rigorously accounts for these attributes, enhancing the credibility of the results.
4. Additionally, the findings by methodologies[21, 22] which positioned alternative A_5 as the second-best option for MWTT, are equally questionable. This option is characterized by harmful emissions, being costly and sustainable energy consumption, all factors that undermining it viability.

5|Conclusion

Hyperbolic fuzzy sets constitutes a novel extension of fuzzy sets, characterized by greater flexibility and independence compared to traditional fuzzy set models. Notably, there has been a lack of entropy measures specifically tailored for HyFSs. To address this gap, an innovative hyperbolic entropy measure have been introduced, along with a new hyperbolic score function that proves to be more logical and reliable than current counterparts. Despite the potential benefits of hybrid methods in hyperbolic fuzzy environment, research in this area has been limited. In response, we present two comprehensive MCDM algorithms : HyF-EM-SWARA-COPRAS and HyF-EM-SWARA-ARAS. The results illustrate that the proposed frameworks effectively address the complexities associated with MWT options, yielding results that are both logical and intuitive for human decision-making.
The selection of an appropriate and effective MWTT has merged as a critical issue in the management of medical waste. This study seeks to addresses the pressing need for an effective method of selecting the optimal MWTT in India. The resulting ranking not only affirms our conclusions but also highlights the superiority of the proposed model. This evidence underscores the necessity for adopting best practices in MWT to promote public health and environmental safety.
Moreover, the HyF-EM-SWARA-COPRAS and HyF-EM-SWARA-ARAS methodologies can be effectively utilized for MCDM challenges. Their application extends to critical areas such as sustainable supplier selection in healthcare, optimizing energy sources for hospitals or even effectively triaging COVID-19 patients. To enhance the effectiveness of the MCDM technique, the additional weight measurement methods AHP, MEREC, CRITIC, BWM and DEMATEL can be incorporated. We can innovate a novel distance measure for HyFSs which can be applied to other MCDM methods such as TOPSIS, EDAS and VIKOR. This strategic approach will significantly elevate the precision and versatility of decision-making processes.

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Appendix

Table 5. Aggregated HyF-DM.

	A_1	A_2	A_3	A_4	A_5
C_1	(0.5746,0.3592)	(0.7251,0.2502)	(0.5468,0.3863)	(0.7251,0.2502)	(0.8017,0.1751)
C_2	(0.6113,0.3268)	(0.5946,0.3440)	(0.5932,0.3379)	(0.5093,0.4229)	(0.7079,0.2545)
C_3	(0.3665,0.5475)	(0.4264,0.5022)	(0.4806,0.4407)	(0.4640,0.4647)	(0.6677,0.2807)
C_4	(0.7277,0.2395)	(0.6022,0.3262)	(0.5713,0.3562)	(0.5466,0.3815)	(0.7196,0.2526)
C_5	(0.6315,0.3112)	(0.6399,0.3029)	(0.6683,0.2723)	(0.8101,0.1712)	(0.6399,0.3029)
C_6	(0.5805,0.3715)	(0.5498,0.3791)	(0.5065,0.4148)	(0.6263,0.3090)	(0.5921,0.3426)
C_7	(0.8262,0.1565)	(0.8725,0.1119)	(0.5355,0.4025)	(0.5612,0.3771)	(0.5748,0.3748)
C_8	(0.7251,0.2502)	(0.6488,0.2995)	(0.6482,0.2993)	(0.8725,0.1119)	(0.8252,0.1508)
C_9	(0.6336,0.3066)	(0.7045,0.2560)	(0.5912,0.3484)	(0.5354,0.4056)	(0.5692,0.3656)
C_{10}	(0.9039,0.0893)	(0.7963,0.1776)	(0.7832,0.1870)	(0.5942,0.3498)	(0.6022,0.3262)

Table 6. The importance of criteria by the DEs.

	E_1	E_2	E_3	E_4	Aggregated HyFNs	Score Values
C_1	MP	F	P	MP	(0.4061,0.5073)	0.2643
C_2	P	VP	F	VP	(0.3710,0.5504)	0.2336
C_3	P	P	F	VP	(0.3897,0.5315)	0.2486
C_4	VG	MG	VG	G	(0.9014,0.0921)	0.8354
C_5	VG	G	VG	G	(0.9086,0.0841)	0.8474
C_6	F	G	G	MP	(0.6663,0.2764)	0.5197
C_7	VG	MG	MG	G	(0.8030,0.1742)	0.6898
C_8	G	G	MG	VG	(0.7783,0.1879)	0.6584
C_9	P	MP	P	VP	(0.3014,0.5892)	0.1898
C_{10}	VP	P	VP	MP	(0.2563,0.6430)	0.1566

Table 7. Subjective weights of criteria by HyF-SWARA technique.

		z_j	k_j	p_j	S_j
C_5	0.8474	-	1	1	0.1264
C_4	0.8354	0.0120	1.0120	0.9881	0.1249
C_7	0.6898	0.1456	1.1456	0.9763	0.1234
C_8	0.6584	0.0314	1.0314	0.9466	0.1197
C_6	0.5197	0.1387	1.1387	0.8313	0.1051
C_1	0.2643	0.2554	1.2554	0.6622	0.0837
C_3	0.2486	0.0157	1.0157	0.6520	0.0825
C_2	0.2336	0.0150	1.0150	0.6424	0.0812
C_9	0.1898	0.0438	1.0438	0.6154	0.0778
C_{10}	0.1566	0.0332	1.0332	0.5956	0.0753

Table 8. Findings of the HyF-EM-SWARA-COPRAS Method.

	A_1	A_2	A_3	A_4	A_5
M_i	(0.5044,0.4632)	(0.4890,0.4709)	(0.4069,0.5432)	(0.4670,0.4937)	(0.4203,0.5313)
$S(M_i)$	0.3356	0.3255	0.2543	0.3030	0.2650
N_i	(0.3367,0.2037)	(0.3452,0.1935)	(0.3110,0.2024)	(0.3260,0.2060)	(0.4551,0.1226)
$S(N_i)$	0.3245	0.3326	0.3118	0.3182	0.4220
Q_i	0.6913	0.6705	0.6245	0.6657	0.5385
U_i	**100**	**96.94**	**90.34**	**96.30**	**77.90**

Table 9. The normalized aggregated HyF-DM (X).

	A_1	A_2	A_3	A_4	A_5
C_1	(0.4254,0.6408)	(0.2749,0.7498)	(0.4532,0.6137)	(0.2749,0.7498)	(0.1983,0.8249)
C_2	(0.3887,0.6732)	(0.4054,0.6560)	(0.4068,0.6621)	(0.4907,0.5771)	(0.2921,0.7455)
C_3	(0.6335,0.4525)	(0.5736,0.4978)	(0.5194,0.5593)	(0.5360,0.5353)	(0.3323,0.7193)
C_4	(0.2723,0.7605)	(0.3978,0.6738)	(0.4287,0.6438)	(0.4535,0.6185)	(0.2804,0.7474)
C_5	(0.6315,0.3112)	(0.6399,0.3029)	(0.6683,0.2723)	(0.8101,0.1712)	(0.6399,0.3029)
C_6	(0.5805,0.3715)	(0.5498,0.3791)	(0.5065,0.4148)	(0.6263,0.3090)	(0.5921,0.3426)
C_7	(0.8262,0.1565)	(0.8725,0.1119)	(0.5355,0.4025)	(0.5612,0.3771)	(0.5748,0.3748)
C_8	(0.7251,0.2502)	(0.6488,0.2995)	(0.5482,0.2993)	(0.8725,0.1119)	(0.8252,0.1508)
C_9	(0.6336,0.3066)	(0.7045,0.2560)	(0.5912,0.3484)	(.5354,0.4056)	(0.5692,0.3656)
C_{10}	(0.9039,0.0893)	(0.7963,0.1776)	(0.7832,0.1870)	(0.5942,0.3498)	(0.6022,0.3262)

Table 10. Findings of the HyF-EM-SWARA-ARAS Method.

	A_1	A_2	A_3	A_4	A_5	X_0
C_1	0.0237	0.0142	0.0259	0.0142	0.0096	0.0259
C_2	0.0230	0.0243	0.0242	0.0318	0.0164	0.0318
C_3	0.0506	0.0433	0.0366	0.0388	0.0202	0.0506
C_4	0.0142	0.0220	0.0244	0.0.0264	0.0148	0.0264
C_5	0.0446	0.0456	0.0496	0.0723	0.0456	0.0723
C_6	0.0409	0.0386	0.0344	0.0474	0.0431	0.0474
C_7	0.0672	0.0797	0.0303	0.0325	0.0333	0.0797
C_8	0.0426	0.0355	0.0354	0.0687	0.0583	0.0687
C_9	0.0440	0.0525	0.0391	0.0334	0.0370	0.0525
C_{10}	0.0749	0.0513	0.0493	0.0296	0.0308	0.0749
E_i	0.4257	0.4070	0.3491	0.3951	0.3091	0.5302
F_i	0.8029	0.7676	0.6584	0.7452	0.5830	